Math 10B with Professor Stankova
Worksheet, Discussion #7; Thursday, 2/14/2019
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1 Induction

1.1 Concepts

1. Mathematical induction allows us to prove a statement for all n. Each induction problem will be of the form: "Let S_n be the statement that (something) is true for any integers $n \geq 1$" where (something) is some mathematical equality. To solve them, there are three steps:

    1. Base Case: Show that the statement is true for the smallest value $n = 1$.
    2. Inductive Step: State that you are assuming the inductive hypothesis (S_n is true for **some** $n \geq 1$). Then, prove that S_{n+1} is true using S_n.
    3. Conclusion: State that by MMI, we conclude that S_n is true **for all** $n \geq 1$.

    All steps must be written in order to get full credit.

1.2 Examples

2. Prove that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for all $n \geq 1$.
3. Prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all $n \geq 0$.

1.3 Problems

4. True False If we want to prove S_n for all $n \geq 10$, then our base case would be $n = 10$.
5. True False When using induction, if we can show that if S_{100} is true, then S_{101} is true, then S_n must be true for all n.
6. True False Instead of assuming S_n is true and showing that S_{n+1} is true, we can instead assume that S_{n-1} is true and prove that S_n is true.
7. Prove that for all $n \geq 1$

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

8. Prove that for all $n \geq 1$
$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

9. Prove that
$$1 + 3 + 9 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$
for all $n \geq 1$.

10. Prove that $6^n - 1$ is divisible by 5 for all $n \geq 1$.

11. Prove that $n^3 + 2n$ is divisible by 3 for all integers $n \geq 0$.

12. Let $\{a_n\}_{n \geq 1}$ be a sequence defined as $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$. Prove that $a_n \leq 2$ for all $n \geq 1$.

13. Prove that $1! \cdot 1 + 2! \cdot 2 + 3! \cdot 3 + \cdots + n! \cdot n = (n + 1)! - 1.$ for all $n \geq 1$.

14. Let $\{a_n\}_{n \geq 1}$ be a sequence defined as $a_1 = 1, a_2 = 5$ and $a_{n+2} = 5a_{n+1} - 6a_n$. Prove that $a_n = 3^n - 2^n$ for all $n \geq 1$.